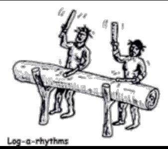
Logarithmic Rules, Ln rules and Exponentials

The key to success at logs is not just knowing your rules, but knowing WHEN TO USE THEM and being good at algebra to simplify/solve after. It is just a process of elimination! If one rule doesn't work ask yourself, can I apply the other? You often have to use multiple rules in one question. The rules work both ways, not only from left to right, hence the ⇔ sign below.



- Logs have the form of base and an argument i.e. $\log_a b$ where $a > 0, b > 0$
- $\log b = \log_{10} b$ (when no base is written it means base 10 by default)

5 Log Rules

Rule 1

Name of Rule: Inverse Rule (aka snail rule)

$$\log_a b = c \Leftrightarrow a^c = b$$

In Words/Pictures and when we use

Words: Logs are just the inverse of indices. This rule turns a log into an index form (gets rid of it) and vice versa. In other words, Any log can be written in index form and vice versa.

Picture: $a^c = b \Leftrightarrow \log_a b$ — "The power you raise a to get c is b"

When/How Do We Use This?

The rule works both ways (from right to left and from left to right)

- Left to right: $\log_a b = c$ is replaced with $a^c = b$ (see examples 3-8)
  - ✓ We use this to turn a log into an index form which is mainly for solving type questions when we want to get rid of the log in order to solve for the unknown (see examples 71 onwards, but don't do these yet)
- Right to left: $a^c = b$ is replaced with $\log_a b = c$
  - ✓ We use this to turn an index form into a log. (see examples 1-2) We very rarely use this UNLESS we are specifically asked to turn an index form into a log. We CAN use it later on (in examples 50 onwards, but don't do these yet), however it will show a nicer method that allow one to appreciate how the 4 solving things link up.

Common Mistakes And Where Students Go Wrong

Failure to realise the following extra cancellation laws resulting from this rule:

- $\log_a 1 = 0$
- $\log_a a = 1$
- $\log_a a^x = x$
- $a^{\log_a x} = x$ (very often missed by students)

(see examples 9-20)

If we apply the rule, the above make sense right?

- $a^0 = 1$ (turning into an index gives this which is true)
- $a^1 = a$ (turning into an index gives this which is true)
- $a^x = a^x$ (turning into an index gives this which is true)
- $\log_a x = \log_a x$ and then turn into an indices

Rule 2

Name: Power

$$c\log_a b \Leftrightarrow \log_a b^c$$

Associated Indices Rule Link

$$(a^b)^c \Leftrightarrow a^{bc}$$

This is the same in terms of the fact we multiply by the power, except with logs we bring the power to the front and multiply it at the front.

In Words/Pictures

Words: We can bring the power up and we can also bring the power down. The rule works both ways.

Picture:

When/How Do We Use This?

The rule works both ways (from right to left and from left to right)

- Left to right: $c\log_a b$ is replaced with $\log_a b^c$
  - ✓ We use this to bring the power up if we want to use rules 3 or 4 below or to get into a certain form (These questions are too easy to come up on their own. Wait until you cover multiplication and division below)
- Right to left: $\log_a b^c$ is replaced with $c\log_a b$
  - We use this to get into a certain form

Common Mistakes And Where Students Go Wrong

This is a really really common mistake!!!! The power rule is for the power of the argument, NOT the power of the entire log. This means the rule works for $\log_a b^c = c\log_a b$

However, $(\log_a b)^c$ is NOT the same as $c\log_a b$

$(\log_a b)^c$ means the entire log raised to the power of c whereas $\log_a b^c$ mean only the argument b is raised to the power of c

Hence: $(\log_a b)^c \neq c\log_a b$

Rule 3

Name Of Rule: Multiplication (aka Product Rule)

$$\log_a bc \Leftrightarrow \log_a b + \log_a c$$

Associated Indices Rule Link

$$a^b a^c \Leftrightarrow a^{b+c}$$

For both indices and logs, multiplication goes to addition, but watch out with logs because it is the opposite - one log multiplied goes to two logs added whereas with indices two bases multiplied go to one base added. But it is the same notion of two things going to one!

In Words/Pictures

Words: One log multiplied can be turned into 2 logs added and vice versa

Picture:

When/How Do We Use This?

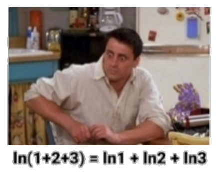
The rule works both ways (from right to left and from left to right)

- Left to right: $\log_a bc$ is replaced with $\log_a b + \log_a c$
  - ✓ We use this to turn one log multiplied into two logs added (see example 21 part i and 27. You can do more examples once you get to division below as they often come up together)
- Right to left: $\log_a b + \log_a c$ is replaced with $\log_a bc$
  - ✓ We use this to turn multiple logs added into one log multiplied. This is used more than the left to right above. (see example 24. You'll can more examples once you get to division below as they often come up together)

Common Mistakes And Where Students Go Wrong

Mistake 1: You can't expand/distribute a log, it is fixed with its argument, just like trig:

$\log(a \pm b) \neq \log a \pm \log b$



ln(1+2+3) = ln1 + ln2 + ln3

Mistake 2: ONE log multiplied goes to TWO added, two logs do NOT go to two logs!

$\log_a b \times \log_a c \neq \log_a b + \log_a c$

Watch out:

- In order to use this rule the coefficients of the logs must be 1
  - $\log_a bc \Leftrightarrow 1\log_a b + 1\log_a c$
  - If they aren't 1, use Power Rule 2 first to bring the power up
- The bases must match. If they don't, use rule 5 (not in your course)

Rule 4

Name Of Rule: Division (aka Quotient Rule)

$$\log_a \frac{b}{c} \Leftrightarrow \log_a b - \log_a c$$

Associated Indices Rule Link

$$\frac{a^b}{a^c} \Leftrightarrow a^{b-c}$$

For both indices and logs, division goes to subtraction, but watch out with logs because it is the opposite - one log divided goes to two logs subtracted whereas with indices two bases divided go to one base subtracted. But it is the same notion of two things going to one!

In Words/Pictures

Words: We can make two logs divided into one log divided and vice versa. This is the same as rule2 except we have division instead of addition.

Picture:

When/How Do We Use This?

The rule works both ways (from right to left and from left to right)

- Left to right: $\log_a \frac{b}{c}$ is replaced with $\log_a b - \log_a c$
  - ✓ We use this to turn one log divided into two logs added (see examples 21 part ii, 22, 23 and 28-30)
- Right to left: $\log_a b - \log_a c$ is replaced with $\log_a \frac{b}{c}$
  - ✓ We use this to turn multiple logs subtracted into one log divided (see examples 25-26 and 72-74) Questions come up a lot with rules 3 and 4!!!!

Common Mistakes And Where Students Go Wrong

There are 3 common mistakes:

Mistake 1	Mistake 2	Mistake 3
$\frac{\log a}{\log b} \neq \log a - \log b$	$\log\frac{a}{b} \neq \log(a-b)$ ONE log divided goes to TWO subtracted, two logs do NOT go to two logs	$\frac{\log a}{\log b} \neq \log\frac{a}{b}$

Watch out In order to use this rule

- The coefficients of the logs must be 1
  - $1\log_a b - 1\log_a c \Leftrightarrow \log_a \frac{b}{c}$
  - If they aren't 1, use Power Rule 2 first to bring the power up
- The bases much match. If they don't, use rule 5 (not in your course)

Rule 5 (NOT in A Level syllabus– included for completeness for IB and math uni entrance exams)

Name Of Rule: Change Of Base

$$\log_a b \Leftrightarrow \frac{\log_c b}{\log_c a}$$

We always use this rule from left to right. This rule is not on the syllabus, but it makes your life easier to know it as you can use if a harder question comes up). It used to be on your syllabus, but got taken off.

In Words/Pictures

Words: You can change the base of a log to anything you want

Picture: $\log_a b = \frac{\log_? b}{\log_? a}$ — your choice

When/How Do We Use This?

This is really easy. The bases of the logs simply do not match (one might have a base 2 and the other might have a base 3 for example) (see examples 36, 37, 78 and 79)

Common Mistakes

This rule is usually applied quite well. Once you have applied it tough ask yourself if any are just numbers and you can apply rule 1

Summary Of All Log Rules

Rule	Description
Rule 1: $\log_a b = c \Leftrightarrow a^c = b$	Rule 1: Any log can be written in index form and vice versa.
Rule 2: $c\log_a b \Leftrightarrow \log_a b^c$	Rule 2: We can bring the power up and down
Rule 3: $\log_a bc \Leftrightarrow \log_a b + \log_a c$	Rule 3: one log multiplied goes to two logs added and vice versa (need to use rule if there is no 1 in front)
Rule 4: $\log_a \frac{b}{c} \Leftrightarrow \log_a b - \log_a c$	Rule 4: one log divided goes to two logs subtracted and vice versa (need to use rule 1 if there is no 1 in front)
Rule 5: $\log_a b \Leftrightarrow \frac{\log_c b}{\log_c a}$ (optional to learn – is not in your syllabus)	Rule 5: We can change the base of any log

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Logs questions are mainly about simplifying questions (most of column 2) versus solving questions (column 3). Graphing and differentiation have also both been included at the very bottom of this column for completeness

Simplifying (there are 6 types)

Type 1: simplifying/evaluating 1 log (use rule 1)

Method for examples 1-2: Use rule 1 from right to left

Method for examples 3-8: Use rule 1 from left to right

- ✓ **Way 1:** If you have a calculator: type these straight in instead (look for the log button with a base which is sometimes hidden). $a^c = b \Leftrightarrow \log_a b$ (equals / raised to the power)
- ✓ **Way 2:** If you don't have a calculator set the given log equal to x and solve for x.

Method for examples 9-20: You can write the answer by using the cancellation laws

Example 1	Example 2	Example 3	Example 4	Example 5	Example 6	Example 7	Example 8
Write $4^3 = 64$ as a logarithm	Write $3^{-2} = \frac{1}{9}$ as a logarithm	Simplify $\log 1000$	Evaluate $\log_2 8$	Simplify $\log_3 1$	Simplify $\log_2 \frac{1}{16}$	Simplify $\log_{25}\frac{125}{\sqrt{5}}$	Simplify $\log_4 8$
$4^3 = 64$ $\log_4 64 = 3$	$3^{-2} = \frac{1}{9}$ $\log_3 \frac{1}{9} = -2$	When no base is written it means base 10 $\log_{10} 1000$ Type this into calc OR set equal to x and use way 1 $\log_{10} 1000 = x$ $10^x = 1000$ $x = 3$	$\log_2 8 = x$ Let's do this way 2 $2^x = 8$ Need all to be base 2 $2^x = 2^3$ Equate the powers $x = 3$	$\log_3 1 = x$ Let's do this way 2 $3^x = 1$ Need all to be base 3 $3^x = 3^0$ Equate the powers $x = 0$	$\log_2 \frac{1}{16} = x$ Let's do this way 2 $2^x = \frac{1}{16}$ Need all to be base 2 $2^x = 2^{-4}$ Equate the powers $x = -4$	$\log_{25}\frac{125}{\sqrt{5}} = x$ Let's do this way 2 $25^x = \frac{5^3}{5^{\frac{1}{2}}}$ $5^{2x} = 5^{\frac{5}{2}}$ $2x = \frac{5}{2}$ $x = \frac{5}{4}$	$\log_4 8 = x$ $4^x = 8$ $(2^2)^x = 2^3$ $2^{2x} = 2^3$ $2x = 3$ $x = \frac{3}{2}$

Example 9	Example 10	Example 11	Example 12	Example 13	Example 14
Simplify $\log_b 1$	Simplify $\log_b b$	Simplify $\log_c c^5$	Simplify $\log_c c^3$	Simplify $\log_b 1$	Simplify $\log_b b$
0	1	5	3	0	1

Example 15	Example 16	Example 17	Example 18	Example 19	Example 20
Simplify $3\log_C C^4$	Simplify $3^{\log_3 7}$	Simplify $a^{\log_a 6}$	Simplify $8^{\log_2 5}$	Simplify $3\log_C C^4$	Simplify $3^{\log_3 7}$
$3\log_C C^4$ $= 3(4)$ $= 12$	$= 7$	$= 6$	$= 2^{3^{\log_2 5}}$ $= 2^{\log_2 5^3}$ $= 5^3$ $= 125$	$3\log_C C^4$ $= 3(4)$ $= 12$	$= 7$

Type 2: Turn 1 log into 2 or more logs (use rules 2/3 &4)

Type 3: Turn 2 or more logs into 1 log (use rules 2/3 & 4)

Example 21	Example 22	Example 23	Example 24	Example 25	Example 26
Given: $\log_2 x = p$ and $\log_2 y = q$. Find: i. $\log_2 x^2y^3$ in terms of p & q; ii. $\log_2 \frac{x^4}{y^2}$ in terms of p & q	Given that: $\log_{10} x = a$, $\log_{10} y = b$ and $\log_{10} z = c$. Find: $\log_{10}\frac{x^2y^4}{\sqrt[3]{z}}$ in terms of a, b & c	Given that $\log_2\frac{32}{8^y}$ can be written as $px + qy$. Find the value of p and q	Simplify $2\log y + 3\log z^2$	Simplify $5\log_3 x^3 - 4\log_3 y^2$	Simplify $2\log_2 12 - 3\log_2 5 - 2\log_2 2$
i. $\log_2 x^2y^3 = \log_2 x^2 + \log_2 y^3 = 2\log_2 x + 3\log_2 y = \mathbf{2p + 3q}$ ii. $\log_2\frac{x^4}{y^2} = \log_2 x^4 - \log_2 y^2 = 4\log_2 x - 2\log_2 y = \mathbf{4p - 2q}$	$= 2\log_{10} x + 4\log_{10} y - \frac{1}{3}\log_{10} z$ $= \mathbf{2a + 4b - \frac{1}{3}c}$	Work on LHS $\log_2\frac{32}{8^y} = \log_2 32 - \log_2 8^y = \log_2 2^5 - \log_2 2^{3y} = 5 - 3y$ Compare to RHS $px + qy$ ∴ $p = 5$ and $q = -3$	When no base is written, it means base 10 (but don't need to write it) $\log y^2 + \log(z^2)^3 = \log y^2 + \log z^6 = \mathbf{\log y^2z^6}$	$= \log_3(x^3)^5 - \log_3(y^2)^4 = \log_3 x^{15} - \log_3 y^8 = \mathbf{\log_3\frac{x^{15}}{y^8}}$	$\log_2 12^2 - \log_2 5^3 - \log_2 2^2 = \log_2 144 - \log_2 125 - \log_2 4 = \log_2 144 - (\log_2 125 + \log_2 4) = \log_2\frac{144}{125 \times 4} = \mathbf{\log_2\frac{36}{125}}$

Type 4: Turn 1 log into multiple logs by adapting numbers (use rule 2/3 and 4)

Example 27	Example 28	Example 29	Example 30
Let: $\log_8 16 = p$. Find in terms of p: i. $\log_8 2$; ii. $\log_8 8q$	Let: $p = \log_a 16$ and $q = \log_a 25$ where a is a constant. Find in terms of p and/or q: i. $\log_a 256$; ii. $\log_a 100$; iii. $\log_a 80 \times \log_a 3.2$	Let: $\log_a 5 = p$ and $\log_a 2 = q$. Find in terms of p and/or q: i. $\log_a 10$; ii. $\log_a 8$; iii. $\log_a 2.5$	Let: $u = \log_a 2$ and $w = \log_a 3$ where $p > 1$. Find: i. $\log_a 6$; ii. $\frac{1}{2}\log_p\left(\frac{27p^4}{4}\right)$

Type 5: Getting into certain forms (use rules 2/3 and 4)

***Exam Tip - Often required to use this to get an answer form at the end of a question ***

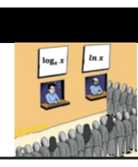
Type 6: Change Of Base (use rule 5)

Not in a level, but in uni entrance exams

Example 31	Example 32	Example 33	Example 34	Example 35	Example 36	Example 37
Write $\log_4 9$ in the form $c\log_2 3$	Write $4\log_3\sqrt{5}$ in the form $\log_a b$	Write $\frac{1}{2}\log 125$ in the form $\log 5^a$	Write $\log_a\left(\frac{1}{x}\right)$ in the form $a\log x$	Write $\frac{\log\left(\frac{1}{1600}\right)}{\log\left(\frac{1}{2}\right)}$ in the form $\frac{\log\left(\frac{a}{k}\right)}{\log b}$	Show that $\log_8 x$ is the same as $\frac{2}{3}\log_4 x$	Given $\log_a 2 = 2$ and $\log_a 5 = y$, find $\log_a 20$ in terms of y
This just wants us to turn the 9 into a 3 and write a number at the front. Using Rule 4 from right to left we get: $\log_4 3^2$ $= \mathbf{2\log_4 3}$	$2\log_3 5$ This now wants us to get rid of the number at the front. Using Rule 4 from left to right we get: $\log_3 5^2 = \mathbf{\log_3 25}$	$= \frac{1}{2}\log 5^3 = \log(5^3)^{\frac{1}{2}} = \mathbf{\log 5^{\frac{3}{2}}}$	$= \log_a 1 - \log_a x = 0 - \log_a x = \mathbf{-\log_a x}$	... $\therefore a = 1600, b = 2$	$\log_4 8$ is just a number (use calc or knowledge from examples 1-6 to help simplify this) $= \frac{\log_4 x}{\frac{3}{2}} = \mathbf{\frac{2}{3}\log_4 x}$	$= \frac{\log_a(4 \times 5)}{\log_a 2} = \frac{\log_a(2^2 \times 5)}{\log_a 2} = \frac{2\log_a 2 + \log_a 5}{\log_a 2} = \frac{2(2) + y}{2} = \mathbf{2 + \frac{1}{2}y}$

Ln and Exponentials

The rules for $\ln(x)$ work the exact same way for $\log(x)$. $\ln(x)$ is just log with a base of e hence $\log_e(x)$. We prefer to write $\ln(x)$! There are 2 extra important things to realise though (although its best to look at the worked examples below to get to grips with this as the words below can be confusing for some)



- e and ln are inverses so cancel each other out when ln is in the power of an exponential

  This should make sense since e takes a power, $e^{\text{something}}$, therefore the ln must be in the power and we can say: $e^{\ln(x)} = x$

- e and ln are inverses so cancel each other out when the e is in the argument of ln

  This should make sense since ln takes an argument, ln something, therefore the e must the argument and we can say: $\ln(e^x) = x$

The ln rules work the same as log rules:

$\ln a + \ln b = \ln a - \ln b$

$\ln a - \ln b = \ln\left(\frac{a}{b}\right)$

$n\ln a = \ln(a^n)$

Also know that

$\ln 1 = 0$

$\ln e = 1$

$\ln e^x = x$

$e^{\ln x} = x$

You normally don't get questions like the examples 29-40 below on their own, but they are good grounding to understand solving

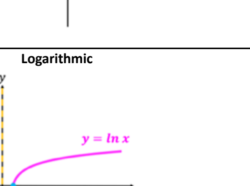
Example 38	Example 39	Example 40	Example 41
Simplify: $e^{\ln(2x)}$	Simplify: $e^{\ln(x^3)}$	Simplify: $e^{3\ln(x)}$	Simplify: $e^{3\ln(x+1)}$
$e^{\ln(2x)}$ Cancel $e^{\ln}$ since they sit next to each other $= \mathbf{2x}$	$e^{\ln(x^3)}$ Cancel $e^{\ln}$ since they sit next to each other $= \mathbf{x^3}$	$e^{3\ln(x)}$ Power Rule: $c\log_a b \Leftrightarrow \log_a b^c$ $= e^{\ln x^3}$ Cancel out the $e^{\ln}$ since they sit next to each other $= \mathbf{x^3}$	$e^{3\ln(x+1)}$ Power Rule: $c\log_a b \Leftrightarrow \log_a b^c$ $= e^{\ln(x+1)^3}$ Cancel out the $e^{\ln}$ since they sit next to each other $= \mathbf{(x+1)^3}$

Example 42	Example 43	Example 44	Example 45
Simplify: $e^{\ln(x)+\ln(y)}$	Simplify: $e^{\ln(2x)+\ln(3y)}$	Simplify: $e^{2\ln(x-1)} + 3$	Simplify: $\ln(e^{3x})$
Multiplication Rule: $\log_a bc \Leftrightarrow \log_a b + \log_a c$ $= e^{\ln(xy)}$ Cancel out the $e^{\ln}$ since they sit next to each other $= \mathbf{xy}$	Multiplication Rule: $\log_a bc \Leftrightarrow \log_a b + \log_a c$ Cancel out the $e^{\ln}$ since they sit next to each other $= 2x \cdot 3y$ $= \mathbf{6xy}$	Power Rule: $c\log_a b \Leftrightarrow \log_a b^c$ $= e^{\ln(x-1)^2} + 3$ Cancel out the $e^{\ln}$ since they sit next to each other $= (x-1)^2 + 3$ $= (x-1)(x-1) + 3$ $= x^2 - x - x + 1 + 3$ $= \mathbf{x^2 - 2x + 4}$	Cancel $\ln(e)$ since they sit next to each other $= \mathbf{3x}$

Example 46	Example 47	Example 48	Example 49
Simplify: $\ln(e^{2x+4})$	Simplify: $\ln(e^{x+y})$	Simplify: $\ln(e^{x+y})^2$	Simplify: $\ln(e^{2x+y})^4$
$\ln(e^{2x+4})$ Cancel $\ln(e)$ since sit next to each other $= \mathbf{2x + 4}$	$\ln(e^{x+y})$ Cancel $\ln(e)$ since sit next to each other $= \mathbf{x + y}$	$\ln(e^{x+y})^2$ Power Rule: $c\log_a b \Leftrightarrow \log_a b^c$ $= 2\ln(e^{x+y})$ Cancel $\ln(e)$ since sit next to eachother $= 2(x+y) = \mathbf{2x + 2y}$	$\ln(e^{2x+y})^4$ Power Rule: $c\log_a b \Leftrightarrow \log_a b^c$ $= 4\ln(e^{2x+y})$ Cancel $\ln(e)$ since sit next to eachother $= 4(2x+y) = \mathbf{8x + 4y}$

Graphs

Exponential



Step 1: Find the y asymptote

This is the value that is added or subtracted to the exponential $y = ae^{bx+c} + d$ gives the asymptote is $y = d$ (it's just the value added or subtracted to the root)

Step 2: Find where graph crosses the x axis

Set $y = 0$ and solve for x (ln both sides to solve. If no solution means no x intercept. Can't take ln of a negative number)

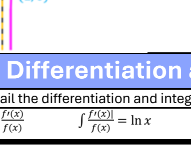
Step 3: Find where graph crosses the y axis

set $x = 0$ and solve for y

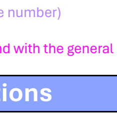
Step 4 : connect the features found above

Keep in mind that we don't touch the asymptote (tend to it) and cross the intercepts found with the general shape of an exponential in mind.

Logarithmic



This is just the inverse of the exponential graph (reflection in the line y=x i.e. swap the x's and y's)



Step 1: Find the x asymptote

x asymptote: we set the argument of ln equal to zero and solve for x.

For example $y = a\ln(bx + c) + d$ we just set $bx + c = 0$ gives x asymptote $x = -\frac{c}{b}$

Step 2: Find where graph crosses the x axis

set $y = 0$ and solve for x (raise e both sides)

Step 3: Find where graph crosses the y axis

set $x = 0$ and solve for y (If no solution means no x intercept – can't have ln of a negative number)

Step 4 : connect the features found above

Keep in mind that we don't touch the asymptote (tend to it) and cross the intercepts found with the general shape of an exponential in mind.

Differentiation and Integration of Logarithmic and Exponential Functions

This is covered in detail the differentiation and integration advanced techniques sheet. This has just been put here as. Reminder that it exists.

$y = \ln f(x) \Rightarrow \frac{dy}{dx} = \frac{f'(x)}{f(x)}$ $\int \frac{f'(x)}{f(x)} = \ln x$ $\int \ln x = \int 1 \times \ln x$ and use parts	$y = e^{f(x)} \Rightarrow \frac{dy}{dx} = f'(x)e^{f(x)}$

Solving (there are 4 types – solving log, ln, exponentials

Each solving type below is covered with examples in the corresponding column (the first time each tip appears is written in green)

Type 1: Solving powers of x and base other than e (start with unknown power)	Type 2: Solving powers of x and base e (start with unknown power)	Type 3: Solving logs start with log and get rid	Type 4: Solving ln start with ln and get rid
Method **If 2 terms:** Log of both sides and use rule 4 **If 3 terms:** Use indices rules and then becomes a hidden quadratic (if have 3 terms). Proceed as above after.	**Method** Take the natural log of both sides	**Method** Use rule $\log_a b = c \Leftrightarrow a^c = b$ If more than one log condense using rules 2/3 If a coefficient in-front use rule 4 first	**Method** Either replace as $\log_e x$ and proceed as normal in type 3 or raise both sides to the power e
Example 50: Power of x (Easy) Solve $5^{x-2} = 7$ Way 1: Log both sides: $\log 5^{x-2} = \log 7$; Use Power rule $(x-2)\log 5 = \log 7$; Expand $(\log 5)x - 2\log 5 = \log 7$ ... Way 2: Use rule 1: $\log_5 7 = x - 2$; Type $\log_5 7$ into your calculator; $1.21 = x - 2$; $x = 3.21$	**Example 60** Solve $e^{3x} = 10$ Ln both sides: $\ln e^{3x} = \ln 10$ e^{ln} cancels $3x = \ln 10$ $\therefore x = \frac{\ln 10}{3}$	**Example 70** Solve $\log_2 x = 3$ We have the form 1 log equal to a number which is what we want Use 'snail rule' $2^3 = x$ $x = 8$	**Example 79** Solve for x: $\ln(x-2) = 5$ Way 1: Raise e both sides: $e^{\ln(x-2)} = e^5$; e^{ln} cancels; $x - 2 = e^5$; $\therefore x = e^5 + 2$ Way 2: Replace with $\log_e x$: $\log_e(x-2) = 5$; Using the Snail Rule: $e^5 = x - 2$; $\therefore x = e^5 + 2$

Example 59
Solve $2(3^{3y}) - 5(3^{2y}) - 9(3^y) + 18 = 0$
$2(3^y)^3 - 5(3^y)^2 - 9(3^y) + 18 = 0$ $(3^y + 2)(3^y - 3)(2(3^y) - 3) = 0$ $3^y \neq -2, 3^y = 3, 3^y = \frac{3}{2}$ $y = 1, y = \frac{\log\left(\frac{3}{2}\right)}{\log 3}$

$e^x = \sqrt{\frac{5}{2}}, e^x \neq -\sqrt{\frac{5}{2}}$

$x = \ln\sqrt{\frac{5}{2}}$

$x = \frac{1}{2}\ln\left(\frac{5}{2}\right)$

$\log_3 x + 5\log_x 3 + 6 = 0$

$\log_3 x + 5\frac{\log_3 3}{\log_3 x} + 6 = 0$

$\log_3 x + 5\frac{1}{\log_3 x} + 6 = 0$

$(\log_3 x)^2 + 5\log_3 x + 6 = 0$

$(\log_3 x + 2)(\log_3 x + 3) = 0$

Put substitution back

$\log_3 x = -2$ and $\log_3 x = -3$

$x = \frac{1}{9}, x = \frac{1}{27}$

Exam Tip : Always remember to check whether your solutions are valid, by plugging your answers into the original question and discount if any are negative. You will lose marks if you don't reject invalid solutions

Summary of solving types/process of elimination – when to do that (when to log, ln , raise to the power of e or snail)

Type 1: Number to power x	Type 2: Exponential to power x	Type 3: Log(s) in equation	Type 4: Ln in equation
number$^{\text{something}}$	$e^{\text{something}}$	log something	ln something
***log* both sides 'Log'-ing**	***ln* both sides 'ln'-ing**	**Turn into one log and Snail 'snailing'**	***e* both sides 'exponential'-ing**
Checklist first: **2 terms or 3 terms?** **2 terms only?** log both sides If there a product use the rule $\log(ab) = \log a + \log b$ Group the x terms as usual and solve **3 terms?** Treat as a hidden quadratic first, get the solutions and and then proceed as above	**Checklist first:** **2 terms or 3 terms?** **2 terms only?** ln both sides If there a product use the rule $\ln(ab) = \ln a + \ln b$ Group the x terms as usual and solve **3 terms?** Treat as a hidden quadratic first, get the solutions and and then proceed as above ln both sides then ln e cancels, So we get, Solve from here	**Checklist first:** **Are there multiple logs?** Need to get 1 log equal to a number first **If more than one log put the logs on the left and use:** $\log a + \log b = \log ab$ $\log a - \log b = \log\frac{a}{b}$ Note: If no 1 infront of the logs bring the power up first Now you should have the form $\log_? ? = number$ Note if stuck: When no base is written replace with 10 Use the snail method	**Checklist first:** **Are there multiple ln's?** Need to get 1 ln equal to a number first using the Multiplication, Division and Power rules. **If more than ln put the $ln's$ on the left and use:** $\ln a + \ln b = \log ab$ $\ln a - \ln b = \ln\frac{a}{b}$ Note: If no 1 infront of the $ln's$ bring the power up first Now you should have the form $\ln ? = number$ Option 1: Raise both sides to power e / Option 2: replace ln with log_e and use snail method raise e both sides then e^{ln} cancels, So we get, Solve from here